

## Decoherence and Planck's Radiation Law

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In the present note the foundations of the theory of environment-induced decoherence are considered. It is pointed out that the common arguments on the diagonalisation of the density matrix are based on questionable hidden assumptions, conflicting with accepted physical results. An alternative interpretation of the phenomena related to decoherence is proposed. We refer to (Joos 1999) and (Zurek 1999) for introductions to the subject of decoherence theory and for background material. An historical perspective of decoherence theory with some relevant critical remarks on the diagonalisation process is provided in (Santos & Escobar, 1999).

We focus on Joos excellent survey of decoherence theory (Joos 1999), whose clarity makes its relatively easy to spot the inconsistencies in the argument. Given a system  $S$  in a superposition of eigenstates  $|n\rangle$  and its environment  $W$  in a state  $\Phi_o$ , the pointer states are identified as those states  $|\Psi(t)\rangle$  in  $W$  resulting from the interaction between  $S$  and  $W$

$$|n\rangle|\Phi_o\rangle \longrightarrow \exp(-iH_{int})|n\rangle|\Phi_o\rangle =: |n\rangle|\Phi_n(t)\rangle.$$

The states  $|\Phi_n(t)\rangle$  result from the entanglement of the environment  $W$  with  $S$  through the interaction Hamiltonian  $H_{int}$  and are usually referred to as the "pointer positions". In this setting the environment  $W$  includes any macroscopic measurement device, which is assumed to be strongly coupled to the rest of the universe. An act of measurement on  $W$  induces a collapse of its state vector into one of the pointer's vector, yielding information about the state of the system  $S$ . The states  $|\Phi_n(t)\rangle$  are described in (Joos 1999) as the states of the "rest of the world". According to decoherence theory the density matrix relative to  $|\Psi(t)\rangle$  is rapidly reduced to a diagonal form, reflecting the system's entanglement with the environment. The off-diagonal interference terms in the density matrix vanish, as superpositions become inaccessible to local observers.

The basic ambiguity underlying this description of the decoherence process stems from the fact that any vector basis can be chosen as a pointer or "preferred" basis, so that the very concept of pointer basis is ambiguous.

Since the environment and any measurement device can be described using an arbitrarily chosen basis  $|\Psi(t)\rangle$ , the "preferred" pointer basis referred to by Joos can only be relative to an observer, as defined by a measurement operator.

It should be clear that the measurement device or the environment do not chose a basis, as physical systems do not chose reference systems. The observer does. The privileged pointer basis is actually determined by the set of possible outcomes of a measurement act performed by an observer. It is the intervention of the observer on the measurement apparatus or on the environment in the course of the measurement process that determines the pointer basis.

An example may clarify the underlying issue. The black body radiation is an instance of macroscopic phenomenon described by a well-understood quantum model. Planck's radiation law

$$\rho(\omega, T) = \frac{1}{\pi^2 c^3} \frac{\hbar \omega^3}{\exp(\hbar \omega / k_B T) - 1}$$

is obtained maximising entropy on discrete energy spectra. In the black body model the evolution of the radiation field is a continuous, reversible process governed by Schroedinger equation that induces the smooth evolution of the system's state vector. On the other hand entropy is maximised on discrete energy spectra. The equilibrium distribution

$$f = \frac{1}{\exp(\hbar \omega / k_B T) - 1}$$

is obtained (Planck 1900 cf. Kuhn 1978, Mackey 1993) maximizing the Boltzmann-Gibbs entropy  $G(f) = - \int f \log f$  on the discrete set  $\epsilon_n = n \hbar \omega$  ;  $n = 0, 1, 2, \dots$ , i.e. on the eigenvalues of the energy operator  $H$ . Planck's law is then obtained as a product of  $f$  and of the mode density  $\omega^2 / \pi^2 c^3$ . Entropy maximisation may be applied to other sets of observables too, but it will yield different results. Entropy maximisation on continuous spectra yields the Jeans-Raleigh law (Einstein 1906, cf. Kuhn 1978). Other observables yield other distribution laws (Mackey 1993).

Planck's radiation law depends on a "preferred basis", i.e on the set of eigenvectors of the energy operator. In other words the Planck distribution is obtained maximizing the entropy of a set of energy measurements, i.e. maximizing the observer's lack of information on the measurement outcomes

of the energy operator. According to decoherence theory the emergence of a "preferred" basis, such as the energy basis in the case of Planck's radiation law, is induced by interaction with the environment. However Planck's radiation law applies also to "closed" systems, so that the emergence of the "preferred basis" relative to the energy operator cannot be attributed to entanglement resulting from interaction with the environment. A concrete example is the cosmic background radiation, which complies with Planck's radiation law in the absence of any external environment, the radiation field being decoupled from the electrically charged matter.

In general the density matrix  $D$  corresponding to  $f$  is

$$D = \frac{e^{-\frac{H}{kT}}}{\text{Trace}(e^{-\frac{H}{kT}})}$$

so that its off-diagonal elements are null in the energy eigenbasis (Von Neumann 1932, V.3). The fact that the off-diagonal elements of  $D$  are null however does not depend on any interaction with the environment, since the system may well be isolated. Actually if the system is isolated its evolution is unitary so that its state  $\Psi$  is a pure state yielding a density matrix  $G = \Psi \otimes \Psi$  with non-null diagonal elements, which however are generally unknown to the observer. The above distinction between  $G$  and  $S$  is essentially the same as that between *type1* and *type2* systems in quantum information theory. (see Pospiech 2000 for a survey).

If we try to interpret this process in terms of environment-induced decoherence we can spot where the key misunderstanding about the meaning of the density matrix arises. The matrix  $D$  where the off-diagonal elements are null just does not represent the state of the system but only encodes the observer's knowledge of the measurement outcomes relative to the energy operator. In the language of quantum measurement theory the matrix  $D$  refers to a mixture. The fact that the off-diagonal elements of the matrix  $D$  are null is hence seen to depend on the observer, as defined by a set of observables or, equivalently, by a measurement operator. The increase the system's entropy just reflects the loss of information of the observer associated to a measurement operator.

It worth remembering that in general the property that a density matrix  $S$  describing a mixture is diagonal with  $\text{Trace}(S) = 1$  encodes only trivial information on the fact that the measurement will yield some result.

Non-trivial diagonal information, i.e. non-trivial information on measurement outcomes, is encoded in the specific values of the diagonal elements. In the case of the black body the macroscopic information determining the values of the diagonal elements is provided by conservation of energy, by the temperature and by the properties of the energy spectrum.

The role of the observer in the decoherence argument is indeed acknowledged in (Joos 1999), as is the fact that the superpositions in the system are not destroyed but merely cease to be identifiable by local observers. However in decoherence theory the pointer basis is implicitly treated as an intrinsic property of the interaction between the system and its environment or a measurement device. This tacit assumption is necessary for the environment-induced decay of the off-diagonal interference terms of the system's density matrix,

$$\rho_S = \sum_{n,m} c_m^* c_n |m\rangle\langle n| \longrightarrow \rho_S = \sum_{n,m} c_m^* c_n \langle \Phi_m | \Phi_n \rangle |m\rangle\langle n|$$

which is then interpreted as the vanishing of superpositions. The assumption however leads to inconsistencies, as shown by the following analysis.

Treating the pointer basis as an intrinsic property of the environment would not matter if the decoherence argument was independent of the chosen pointer basis. However this is not the case. According to the argument in (Joos 1999) and (Zurek 1993), the decoherence process induces the decay of the off-diagonal elements of the systems density matrix,

$$\rho_S \longrightarrow \sum_n |c_n|^2 |n\rangle\langle n|$$

which is interpreted as the emergence of a set of stable macroscopic states. The density matrix however is defined in terms of the pointer basis. Different pointer basis lead to different density matrices for the same state vectors. It is immediate to see that the decoherence process, i.e. the decay of the off diagonal terms in the density matrix, does not commute with a change of basis. Indeed given a density matrix  $A$ , let  $C$  be a change of basis and  $C^{-1}$  its inverse and  $D$  the operator that equates to null the off-diagonal elements. Then

$$DA \neq (C^{-1}DC)A$$

so that the result of the decoherence process depends on the pointer basis, which is selected by the observer and is independent of the underlying physical process. The states associated with a diagonal density matrix in one basis

describe superpositions in the other basis. Indeed any two non-commuting operators induce pointer basis for which the above inequality holds, so that the physical process inducing the diagonalisation appears to depend on the chosen basis. This is absurd, unless one accepts that the diagonal matrix describes an observer-dependent mixture, for which the above argument does not hold. The root of the mistake is the attempt to "objectify" the observer's loss of information, attributing it to a physical process unrelated to the observer.

The above indicates that the result of the entropy maximisation process depends on the observer and that it applies to the measurement outcomes relative to the observer's measurement operator. If our interpretation is correct there must be a flaw in the argument tying the decay of the off-diagonal elements of the density matrix to the interaction with the environment.

The flaw is not hard to find. If one examines the argument leading to the diagonalisation of the system's density matrix, one discovers that it is based on unphysical no-recoil assumptions on the scattering process (Joos 1999), i.e. on ignoring back-action on the environment either directly or through appropriately chosen cut-offs (cf. Unruh & Zurek 1989) or through selective application of fine/coarse graining to different variables (Brun 1993, cf. Feynman & Vernon 1963). It may be noted that the fine/coarse graining approach reveals the role of the observer, which was later fudged by uncritical use of the original results. Under the no-recoil assumption every scattering event multiplies the off-diagonal elements of the local density matrix by a factor  $1 - \epsilon$  (Joos 1999, 3.1.2). This hammers the non-diagonal elements into converging to zero, while preventing the environment from eroding the diagonal elements. The no-recoil assumption forces the density matrix into a very singular form, where the off-diagonal terms converge rapidly to zero, while the diagonal terms remains intact. Applying the no-recoil assumption to a different basis however leads to a diagonal matrix describing a different physical state and which is not diagonal under a change of basis, as shown above.

On the other hand, as shown by the Planck's radiation law, a diagonal matrix referring to a mixture is naturally associated to the system, not on the basis of any physical interaction with the environment, but simply on the basis of entropy maximisation of the mixture relative to a measurement operator. Such entropy maximisation yields the system's macroscopic properties relative to the observer associated to the operator.

The decoherence process reflects then the observer's loss of information, not only on superpositions, but on the microscopic state of the system. The special status of superpositions is indeed spurious, since it depends on the measurement operator being considered, i.e. on the observer. The singling out of superpositions, i.e. of off-diagonal elements of the local density matrix, for special destructive treatment appears as an artefact, based on unphysical assumptions and on confusion between *type1* with *type2* systems, i.e. on attributing pure states' properties to mixtures.

We wrap up our considerations with a simple "Schroedinger's cat" example, illustrating the constraints of global unitarity on local observers. Consider the situation

$$System = Cat, \quad Environment = Rest\ of\ the\ World.$$

and the basis  $A = (|alive\rangle, |dead\rangle)$ . The system's initial state is  $1/\sqrt{2}(|alive\rangle + |dead\rangle)$ . We may consider the system in the basis

$$B = (B_1, B_2) = (1/\sqrt{2}(|alive\rangle + |dead\rangle), 1/\sqrt{2}(|alive\rangle - |dead\rangle)).$$

A change of basis does not affect the state of the system, as long as no basis-dependent measurement takes place. As long as the Cat is not observed, the universe's state-vector, whose evolution is unitary, is

$$B_1^{universe} = 1/\sqrt{2}(|alive^{universe}(t)\rangle + |dead^{universe}(t)\rangle).$$

The phase-related information about the Cat-superpositions is encoded in  $B_1^{universe}(t)$  and it may not be accessible to a basis- $A$ -observer in the state-of-the-Cat subsystem, which can be represented either as a mixture by a diagonal matrix (*type2* system) reflecting the observer's ignorance in a specific basis, or as non-diagonal density matrix with unknown non-diagonal elements (*type1* system). For basis- $A$ -observers the Cat will either die or live once the Cat-subsystem is projected onto that basis by an act of observation/measurement. For an hypothetical observer in basis  $B$  however there are no superpositions. The state-of-the-Cat density-matrix in basis  $B$  is just

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and the outcome of a basis- $B$ -measurement is certain. Such tilted-basis measurements are actually at the core of the Elitzur-Vaidman scheme (Elitzur & Vaidman 1993), where information is extracted from a system without inducing collapse in the "usual" basis.

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